Creep rupture of viscoelastic fiber bundles

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We study the creep rupture of bundles of viscoelastic fibers occurring under uniaxial constant tensile loading. A fiber bundle model is introduced that combines the viscoelastic constitutive behavior and the strain controlled breaking of fibers. Analytical and numerical calculations showed that above a critical external load the deformation of the system monotonically increases in time resulting in global failure at a finite time t_f, while below the critical load the deformation tends to a constant value giving rise to an infinite lifetime. Our studies revealed that the nature of the transition between the two regimes, i.e., the behavior of t_f at the critical load σ_c, strongly depends on the range of load sharing: for global load sharing t_f has a power law divergence at σ_c with a universal exponent of 0.5, however, for local load sharing the transition becomes abrupt: at the critical load t_f jumps to a finite value, analogous to second- and first-order phase transitions, respectively. The acoustic response of the bundle during creep is also studied.

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Fiber reinforced composites are of great technological importance due to their very good performance under extreme circumstances. Under high steady stresses these fiber composites may exhibit time-dependent failure called creep rupture, which limits their life time and consequently has a high impact on the applicability of these materials for construction elements. Both natural fiber composites such as wood [1,2] and various types of fiber reinforced composites [3–5] show creep rupture phenomena, which have attracted continuous theoretical and experimental interest over the past years. The underlying microscopic failure mechanism of creep rupture is very complex depending on several characteristics of the specific types of materials, and is far from being well understood. One of the most important aspects of creep rupture is the statistics of life time (or time to failure) as a function of the external steady load, however, only a limited number of systematic experimental works is available for fiber reinforced composites [3–5], more information has been accumulated about natural fiber composites [1,2]. In Ref. [6] a theoretical model of creep rupture of brittle matrix composites reinforced with time-dependent fibers was worked out in the spirit of the classical model of Coleman [7]. For natural fiber composites a so-called damage accumulation model has been developed, which simply assumes that the time derivative of the accumulated damage depends exponentially on the external load history of the specimen [8].

In the present paper we study the creep rupture of fiber composites where the fibers have viscoelastic behavior and the microscopic damage mechanism leading to creep rupture is the strain-dependent breaking of fibers under the time evolution of the deformation of the system. Creep failure tests are usually performed under uniaxial tensile loading when the specimen is subjected either to a constant load σ_0 or to an increasing load (ramp loading) and the time evolution of the damage process is followed by recording the strain ε of the specimen and the acoustic signals emitted by microscopic failure events. In the present study we focus on the general aspects of creep rupture, i.e., the behavior of the life time of the bundle as a function of the external load, its dependence on the range of load redistribution, furthermore, general aspects of the acoustic response of the bundle are considered without fitting the theoretical results to any specific materials.

In order to work out a theoretical description of creep failure of viscoelastic fiber composites, we improve the classical fiber bundle model [7,9] that has proven very successful in the study of fracture of disordered materials [10–19]. Our model consists of N parallel fibers having viscoelastic constitutive behavior. For simplicity, the pure viscoelastic behavior of fibers is modeled by a Kelvin-Voigt element that consists of a spring and a dashpot in parallel and results in the constitutive equation $\sigma_0=\beta\dot{\varepsilon}+E\varepsilon$, where β denotes the damping coefficient, and E the Young modulus of fibers, respectively. This equation provides the time-dependent deformation $\varepsilon(t)$ of a fiber at a fixed external load σ_0,

$$\varepsilon(t)=\frac{\sigma_0}{E}[1-e^{-Et/\beta}]+\varepsilon_0 e^{-Et/\beta}, \tag{1}$$

where ε_0 denotes the initial strain at $t=0$. It can be seen that $\varepsilon(t)$ converges to σ_0/E for $t\to\infty$, which implies that the asymptotic strain fulfills Hook's law.

If no fiber failure occurs Eq. (1) would fully describe the time evolution of the system. Motivated by the experimental observations of the acoustic response [21] of fiber composites during creep, we introduce a strain controlled failure criterion to incorporate damage in the model: a fiber fails during the time evolution of the system if its strain exceeds a damage threshold ε_d, which is an independent identically distributed random variable of fibers with probability density $p(\varepsilon_d)$ and cumulative distribution $P(\varepsilon_d)=\int_0^{\varepsilon_d}p(x)dx$. Similar strain controlled breaking was recently used in Ref. [20]. Due to the validity of Hook's law for the asymptotic strain values, the formulation of the failure criterion in terms of strain instead of stress implies that under a certain steady

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load the same amount of damage occurs as in the case of stress controlled failure, however, the breaking of fibers is not instantaneous but distributed over time. When a fiber fails its load has to be redistributed to the intact fibers. As the simplest approach, we assume global load sharing [10,15–19], i.e., after a failure event the excess load is equally distributed among the intact fibers, and hence, at a certain strain ε the load on the surviving fibers of the number $N_s(\varepsilon)$ can be cast into the form $\sigma(\varepsilon)=\sigma_0 N/N_s(\varepsilon)=\sigma_0/[1-P(\varepsilon)]$. The time evolution of the system under a steady external load σ_0 is finally described by the equation,

$$\frac{\sigma_0}{1-P(\varepsilon)}=\beta\dot{\varepsilon}+E\varepsilon, \tag{2}$$

where the viscoelastic behavior of fibers is coupled with the failure of fibers in a global load sharing framework.

For the behavior of the solutions of Eq. (2) two distinct regimes can be distinguished depending on the value of the external load σ_0. When σ_0 is below a critical value σ_c Eq. (2) has a stationary solution ε_s, which can be obtained by setting $\dot{\varepsilon}=0$,

$$\sigma_0=E\varepsilon_s|1-P(\varepsilon_s)|. \tag{3}$$

It means that until this equation can be solved for ε_s at a given external load σ_0, the solution $\varepsilon(t)$ of Eq. (2) converges to ε_s when $t\to\infty$, and no macroscopic failure occurs. However, when σ_0 exceeds the critical value σ_c no stationary solution exists, furthermore, $\dot{\varepsilon}$ remains always positive, which implies that for $\sigma>\sigma_c$ the strain of the system $\varepsilon(t)$ monotonically increases until the system fails globally at a time t_f.

In the regime $\sigma_0\leqslant\sigma_c$ Eq. (3) also provides the asymptotic constitutive behavior of the fiber bundle that can be measured by controlling the external load σ_0 and letting the system relax to ε_s. It follows from the above argument that the critical value of the load σ_c is the static fracture strength of the bundle that can be determined from Eq. (3) as $\sigma_c=E\varepsilon_c[1-P(\varepsilon_c)]$, where ε_c is the solution of the equation $d\sigma_0/d\varepsilon_s|_{\varepsilon_c}=0$, as shown by Sornette [15]. Since $\sigma_0(\varepsilon_s)$ has a maximum of the value σ_c at ε_c, in the vicinity of ε_c it can be approximated as

$$\sigma_0\approx\sigma_c-A(\varepsilon_c-\varepsilon_s)^2, \tag{4}$$

where the multiplication factor A depends on the probability distribution P. A complete description of the system can be obtained by solving the differential equation (2). After separation of variables the integral arises

$$t=\beta\int d\varepsilon\,\frac{1-P(\varepsilon)}{\sigma_0-E\varepsilon[1-P(\varepsilon)]}+C, \tag{5}$$

where the integration constant C is determined by the initial condition $\varepsilon(t=0)=0$.

The creep rupture of the viscoelastic bundle can be interpreted so that for $\sigma_0\leqslant\sigma_c$ the lifetime (or the time to failure) of the bundle is infinite $t_f=\infty$, while above the critical load $\sigma_0>\sigma_c$ global failure occurs at a finite time t_f, which can be determined by evaluating the integral Eq. (5) over the whole domain of definition of $P(\varepsilon)$. From the theoretical and experimental point of view it is very important how t_f depends on the external load above σ_c. When σ_0 is in the vicinity of σ_c, i.e., $\sigma_0=\sigma_c+\Delta\sigma_0$, where $\Delta\sigma_0\ll\sigma_c$, it can be expected that the curve of $\varepsilon(t)$ falls very close to ε_c for a very long time and the breaking of the system occurs suddenly. Hence, the total time to failure, i.e., the integral in Eq. (5), is dominated by the region close to ε_c when $\Delta\sigma_0$ is small. Making use of the power series expansion Eq. (4) the integral in Eq. (5) can be rewritten as

$$t_f\sim\beta\int d\varepsilon\,\frac{1-P(\varepsilon)}{\Delta\sigma_0-A(\varepsilon_c-\varepsilon)^2}, \tag{6}$$

which has to be evaluated over a small ε interval in the vicinity of ε_c. After performing the integration it follows

$$t_f\approx(\sigma_0-\sigma_c)^{-1/2} \quad \text{for} \quad \sigma_0>\sigma_c. \tag{7}$$

Thus, t_f has a power law divergence at σ_c with a universal exponent $-\frac{1}{2}$ independent of the specific form of the disorder distribution $p(\varepsilon)$.

For the purpose of explicit calculations we considered the case of a uniform distribution of the damage thresholds between 0 and a maximum value ε_m, thus, $p(\varepsilon_d)=1/\varepsilon_m$ and $P(\varepsilon_d)=\varepsilon_d/\varepsilon_m$. The stationary solution, the critical load, and the corresponding critical strain can be obtained as $\sigma_0=E\varepsilon[1-\varepsilon/\varepsilon_m]$, $\sigma_c=E\varepsilon_m/4$, $\varepsilon_c=\varepsilon_m/2$, respectively. Finally, the solution of the integral Eq. (5) taking the initial condition also into account can be cast into the implicit form

$$t=-\frac{\beta}{2E}\left\{\frac{1}{\sqrt{1-\dfrac{4\sigma_0}{E\varepsilon_m}}}\ln\frac{\dfrac{\varepsilon}{\varepsilon_m}\left[1+\sqrt{1-\dfrac{4\sigma_0}{E\varepsilon_m}}\right]+\dfrac{2\sigma_0}{E\varepsilon_m}}{\dfrac{\varepsilon}{\varepsilon_m}\left[1-\sqrt{1-\dfrac{4\sigma_0}{E\varepsilon_m}}\right]+\dfrac{2\sigma_0}{E\varepsilon_m}} -\ln\frac{E\varepsilon^2-E\varepsilon_m\varepsilon+\sigma_0\varepsilon_m}{\sigma_0\varepsilon_m}\right\} \tag{8}$$

for $\sigma_0<\sigma_c$ (below the critical point), and

$$t=\frac{\beta}{E}\left\{\frac{1}{\sqrt{\dfrac{4\sigma_0}{E\varepsilon_m}-1}}\left[\arctan\frac{2\dfrac{\varepsilon}{\varepsilon_m}-1}{\sqrt{\dfrac{4\sigma_0}{E\varepsilon_m}-1}} -\arctan\frac{-1}{\sqrt{\dfrac{4\sigma_0}{E\varepsilon_m}-1}}\right]-\frac{1}{2}\ln\frac{E\varepsilon^2-E\varepsilon_m\varepsilon+\sigma_0\varepsilon_m}{\sigma_0\varepsilon_m}\right\} \tag{9}$$

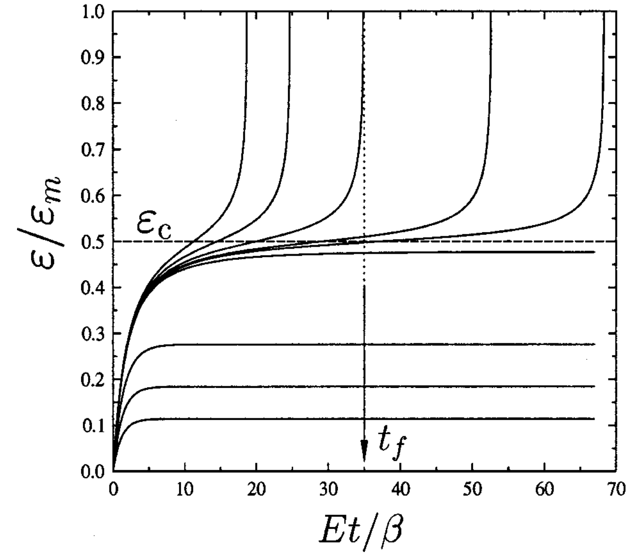

FIG. 1. The analytic solution $\varepsilon(t)$ given by Eqs. (8) and (9) for several values of σ_0 below and above σ_c. The critical strain ε_c and the time to failure t_f for one example are indicated.

for $\sigma_0 > \sigma_c$ (above the *critical point*). The behavior of this analytic solution is illustrated in Fig. 1 for several different values of σ_0.

The time to failure t_f can be determined by setting $\varepsilon = \varepsilon_m$ in Eq. (9), which results in the form

$$t_f \approx \frac{\beta\pi}{2}\sqrt{\frac{\varepsilon_m}{E}}(\sigma_0-\sigma_c)^{-1/2}, \tag{10}$$

in accordance with the above general arguments.

A further important general property of $\varepsilon(t)$ that can be deduced from Eqs. (2) and (5) is that at the time to failure t_f the deformation rate $d\varepsilon/dt$ diverges. For disorder distributions $P(\varepsilon)$ defined in a finite interval the exponent is universal $d\varepsilon/dt \approx (t_f - t)^{-1/2}$.

In order to obtain information about the gradual breaking of fibers during the creep process, in the experiments the acoustic signal emitted by breaking events in a short time interval is investigated. In our fiber bundle model the number of fibers $N_b(t)$ that have been broken up to time t can be determined as $N_b(t) = NP[\varepsilon(t)]$, and hence, its derivative provides the quantity

$$\frac{1}{N}\frac{\partial N_b}{\partial t} = \frac{dP}{d\varepsilon}\frac{d\varepsilon}{dt} = \frac{p(\varepsilon)E\varepsilon}{\beta}\left[\frac{\sigma_0}{E\varepsilon|1-P(\varepsilon)|}-1\right], \tag{11}$$

which is a measure for the acoustic response. The behavior of Eq. (11) for the uniform distribution is illustrated in Fig. 2, where it can be observed that the acoustic activity, i.e., fiber breaking, practically disappears in the plateau region of $\varepsilon(t)$ (compare to Fig. 1), however, it diverges at t_f due to the diverging deformation rate.

Since during a creep test $\varepsilon(t)$ is monitored from which $d\varepsilon/dt$ can be calculated, furthermore, $\partial N_b/\partial t$ is measured

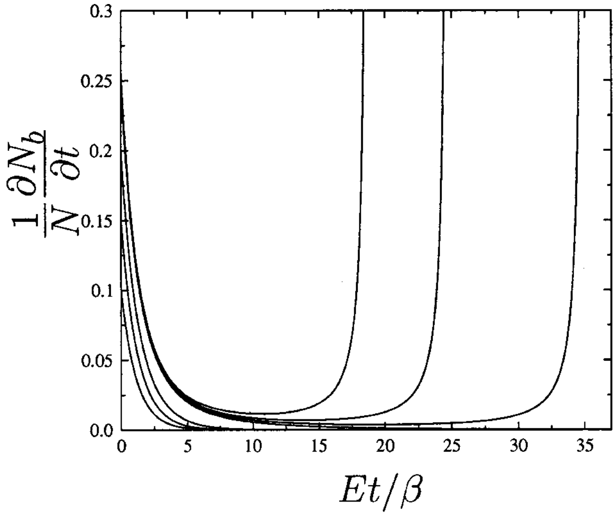

FIG. 2. The analytic solution for the breaking rate Eq. (11) for several values of σ_0 below and above σ_c.

by means of acoustic emission techniques, Eq. (11) makes possible to determine experimentally the distribution of the failure thresholds $p(\varepsilon_d)$.

To complement the predictions of the analytic approach Monte Carlo simulations of the failure process have been performed using global load sharing (GLS) and local load sharing (LLS) for the stress redistribution. The GLS simulation of the creep failure process of a bundle of N fibers proceeds as follows: (i) random breaking thresholds ε_i, $i = 1,\ldots,N$ were chosen according to a probability distribution p, then the thresholds were put into increasing order. (ii) Since the fibers break one by one, the actual load on the fibers after the failure of i fibers is $\sigma_i = \sigma_0 N/(N-i)$, where $i = 0,\ldots, N-1$, and the time between the breaking of the ith and $(i+1\text{th})$ fibers reads as $t_i = -(\beta/E)\ln([\varepsilon_{i+1} - (\sigma_i/E)]/[\varepsilon_i - (\sigma_i/E)])$. (iii) Finally, the time as a function of ϵ can be obtained as $t(\varepsilon_i) = \Sigma_{j=0}^{i} t_j(\varepsilon_j)$ from which $\varepsilon_i(t)$ can be determined. The time to failure t_f of a finite bundle is defined as the time of the failure of the last fiber. To test the validity of the power law behavior of t_f given by Eq. (7) simulations were performed with various distributions in the framework of GLS. The results are presented in Fig. 3 where an excellent agreement of the simulations and the analytic results can be observed. The macroscopic strain of the system $\varepsilon(t)$ and the acoustic response obtained by simulations was also found to be in agreement with the analytic results.

To clarify how the damage process and the behavior of t_f is affected by the range of interaction among fibers, i.e., by the range of load sharing we performed simulations with LLS on a square lattice of 200×200 sites, redistributing the load of the failed fiber on its nearest neighbors. The critical load σ_c was first determined as the static fracture strength of a dry fiber bundle with LLS assuming perfectly elastic behavior for the fibers. Comparing the results of the LLS simulations to the global load sharing results it was observed that above σ_c the failure of the viscoelastic bundle occurs much more abruptly than in the case of GLS.

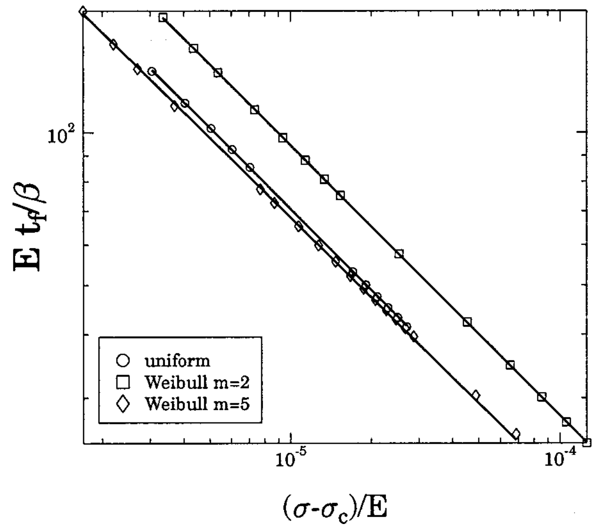

FIG. 3. The behavior of the time to failure t_f for uniform and Weibull distributions with two different Weibull moduli for the GLS case. All the three curves are parallel to each other on a double logarithmic plot with an exponent close to 0.5 in agreement with the general result Eq. (7).

Varying σ as a control parameter the two regimes of the creep rupture process are characterized by an infinite life time below σ_c and by a finite one above the critical point. The nature of the transition between the regimes in the global and local load sharing models can be characterized by studying $1/t_f$ as a function of the control parameter σ. In Fig. 4 it can be observed that below the critical point, when no global failure occurs, $1/t_f$ is zero, while above σ_c it takes a finite value for both LLS and GLS. However, the behavior of $1/t_f$ in the vicinity of σ_c is completely different in the two cases, for GLS the transition is continuous, while for LLS $1/t_f$ has a finite jump, analogously to a second and first order phase transition, respectively.

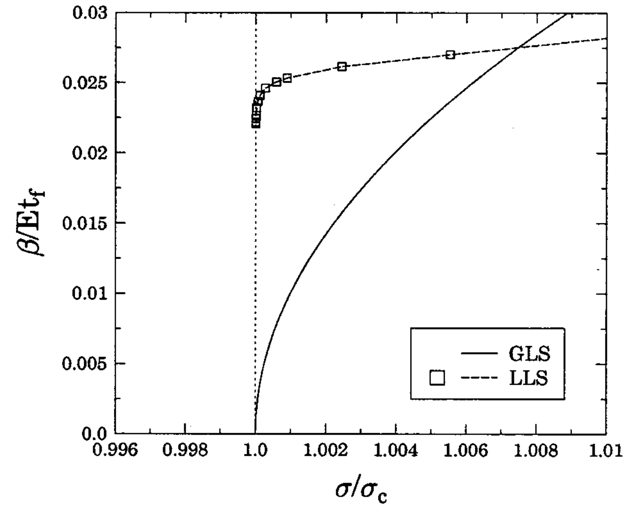

FIG. 4. Comparison of $1/t_f$ for the LLS and GLS cases as a function of the external load.

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